Solutions to Problem 1.

a. When there are n customers in the shop, customers are lost at a rate of $20(n/3)$ customers per hour. Therefore,

$$\begin{aligned}\text{Lost customers / hour} &= 20\left(\frac{0}{3}\right)\pi_0 + 20\left(\frac{1}{3}\right)\pi_1 + 20\left(\frac{2}{3}\right)\pi_2 + 20\left(\frac{3}{3}\right)\pi_3 \\ &= 0\left(\frac{9}{67}\right) + \frac{20}{3}\left(\frac{18}{67}\right) + \frac{40}{3}\left(\frac{24}{67}\right) + 20\left(\frac{16}{67}\right) \\ &\approx 11.34\end{aligned}$$

b. $$\begin{aligned}\text{Expected profit / hour} &= (\text{Expected number of customers / hour})(\text{revenue / customer}) - (\text{cost / hour}) \\ &= \lambda_{\text{eff}}(2) - 4 \\ &\approx (8.6567)(2) - 4 \\ &\approx 13.31\end{aligned}$$

Solutions to Problem 2.

a.
- **State space.** $\mathcal{M} = \{0, 1, 2, \dots\}$
  Each state represents the number of patients in the urgent care center.
- **Arrival rates.** $\lambda_i = \begin{cases} 2 & \text{for } i = 0,1,2,3 \\ 0 & \text{for } i = 4,5,\dots \end{cases}$
- **Service rates.** $\mu_i = \begin{cases} 2 & \text{for } i = 1 \\ 4 & \text{for } i = 2,3,\dots \end{cases}$

b.

$$\left.\begin{aligned} d_0 &= 1 \\ d_1 &= \frac{\lambda_0}{\mu_1} = 1 \\ d_2 &= d_1\frac{\lambda_1}{\mu_2} = 1\left(\frac{2}{4}\right) = \frac{1}{2} \\ d_3 &= d_2\frac{\lambda_2}{\mu_3} = \frac{1}{2}\left(\frac{2}{4}\right) = \frac{1}{4} \\ d_4 &= d_3\frac{\lambda_3}{\mu_4} = \frac{1}{4}\left(\frac{2}{4}\right) = \frac{1}{8} \\ d_5 &= d_4\frac{\lambda_4}{\mu_5} = 0 \\ \Rightarrow d_j &= 0 \quad \text{for } j = 5,6,\dots \\ \Rightarrow D &= \sum_{j=0}^{\infty} d_j = \frac{23}{8} \end{aligned}\right\} \Longrightarrow \left\{\begin{aligned} \pi_0 &= \frac{d_0}{D} = \frac{8}{23} \\ \pi_1 &= \frac{d_1}{D} = \frac{8}{23} \\ \pi_2 &= \frac{d_2}{D} = \frac{4}{23} \\ \pi_3 &= \frac{d_3}{D} = \frac{2}{23} \\ \pi_4 &= \frac{d_4}{D} = \frac{1}{23} \\ \pi_j &= \frac{d_j}{D} = 0 \quad \text{for } j = 5,6,\dots \end{aligned}\right.$$

c. $\ell_q = \sum_{n=s+1}^{\infty} (n-s)\pi_n = (3-2)\pi_3 + (4-2)\pi_4 = \frac{4}{23}$ customers

d. $\lambda_{\text{eff}} = \sum_{i=0}^{\infty} \lambda_i \pi_i = 2\pi_0 + 2\pi_1 + 2\pi_2 + 2\pi_3 + 0\pi_4 = \frac{44}{23}$ customers / hour $\Rightarrow$ $w_q = \frac{\ell_q}{\lambda_{\text{eff}}} = \frac{4}{44} = \frac{1}{11}$ hours

e. Fraction of arriving customers going to Gaussville $= \pi_4 = \frac{1}{23}$

Solutions to Problem 3.

a.

i	$d_i = d_{i-1}(\frac{\lambda_{i-1}}{\mu_i})$	$\pi_i = \frac{d_i}{D}$
0	1	0.0924
1	1.5	0.1386
2	2.25	0.2079
3	1.6875	0.1559
4	1.2656	0.1169
5	0.9492	0.0877
6	0.7119	0.0658
7	0.5339	0.0493
8	0.4005	0.0370
9	0.3003	0.0277
10	0.2253	0.0208
> 10	0	0
	$D = \sum_{i=0}^{\infty} d_i = 10.8242$	

The second agent is on duty in states $3, 4, 5, \ldots$. So, the fraction of time that the second agent is on duty is $1 - (\pi_0 + \pi_1 + \pi_2) \approx 0.5611$.

b. The average length of the queue is

$$
\begin{aligned}
\ell_q &= 0\pi_0 + 0\pi_1 + 1\pi_2 + 1\pi_3 + 2\pi_4 + 3\pi_5 + 4\pi_6 + 5\pi_7 + 6\pi_8 + 7\pi_9 + 8\pi_{10} \\
&= 0(0.0924) + 0(0.1386) + 1(0.2079) + 1(0.1559) + 2(0.1169) + 3(0.0877) \\
&\quad + 4(0.0658) + 5(0.0493) + 6(0.0370) + 7(0.0277) + 8(0.0208) \approx 1.9478
\end{aligned}
$$